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Universal Engineering Model for Cooling Towers

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ABSTRACT
This paper presents a universal engineering model, which can be used to formulates both counter-flow and cross-flow cooling towers. By using fundamental laws of mass and energy balance, the effectiveness of heat exchange is approximated by a second order polynomial equation. Gauss -Newton and Levenberg-Marquardt methods are then used to determine the coefficients from manufactures data. Compared with the existing models, the new model has two main advantages: (1) As the engineering model is derived from engineering perspective, it involves fewer input variables and has better description of the cooling tower operation; (2) There is no iterative computation required, this feature is very important for online optimization of cooling tower performance. Although the model is simple, the results are very accurate. Application examples are given to compare the proposed model with commonly used models.
Keywords: Counter-flow, Cooling Tower, Effectiveness, HVAC

I. INTRODUCTION

Cooling towers are commonly used to dissipate heat from heat sources to heat sink (ambient environment). Their applications are typically in Heating Ventilation and Air Conditioning (HVAC) systems and power generators, etc. Heat rejection of cooling towers is accomplished by heat and mass transfer between hot water droplets and ambient air. Although cooling towers are relatively inexpensive and normally consume around ten percent of the whole system energy, their operation has significant effect to the energy consumption of other related subsystems (RMIRA 1995; Michel 1995). Therefore, optimizing cooling tower performance will not only increase the tower efficiencies but also has direct effect to other subsystems. As such, there has been some research interest in this area. Austin (1997) recommended regression methods to create the models of each component in air conditioning systems for predicting and optimizing the system performance. Flake (1997) utilized a different regression technique to determine parameters of the cooling tower model developed by Braun (1989) and to build a predictive model for optimal supervisory control strategies. However, due to the lacking of an effective and precise model for cooling towers, which is essential to estimate and verify the energy savings by different optimization strategies, the research on optimization of cooling tower performance is still in the primary stage Attempts to develop the cooling tower models have a relative long history, the first such work may trace back to 1925, when Merkel developed a practical model for cooling tower operation, which has been the basis for most modern cooling tower analyses. In his model, the water loss of evaporation is neglected and the Lewis number is assumed to be one in order to simplify the analysis. However, as evaporate water cannot be neglected in cooling tower operation, Merkel's model is not accurate enough and not suitable for real applications. A more rigorous analysis of a cooling tower model that relaxed Merkel's restriction was given by Sutherland (1983). In 1989, Braun developed "effectiveness models" for cooling towers, which utilized the assumption of a linearized air saturation enthalpy and the modified definition of number of transfer units. The models were useful for both design and system simulation and has been adopted by the simulation software TRNSYS (SEL 2000). However, Braun's model needs iterative computation to obtain the output results and is not suitable for online optimization. Bernier (1994) reviewed the heat and mass transfer process in cooling towers at water droplet level and analyzed an idealized spray-type tower in one-dimension, which is useful for cooling tower designers, but no much information is provided to plant operators. Soylemez (1999) presented a quick method for estimating the size and performance of forced draft countercurrent cooling towers and experimental results were used to validate the prediction formulation. Unfortunately, this model also need iterative computation and not suitable for online optimization.

In this paper, a universal engineering model, which can be used to formulate both counterflow and crossflow cooling towers, is proposed. Extending the methods provided by Merkel and Braun and using fundamental laws of mass and energy balance, the effectiveness of heat exchange is approximated by a second order polynomial equation. Gauss-Newton and Levenberg-Marquardt methods are then used to determine the coefficients from manufactures data. Compared with the existing models, the new model has two main advantages: (1) As the engineering model is derived from engineering perspective, it involves fewer input variables and has better description of the cooling tower operation; (2) There is no iterative computation required, this feature is very important for online optimization of cooling tower performance. Although the model is simple, the results are very accurate. Application examples are given for both counter flow and cross flow to compare the proposed model with commonly used models four governing equations can be used to express the mass and energy balance in the system:

1) Mass conservation of air: $m_{a,i} + m_e = m_{a,o}$ (1)
2) Heat conservation of air: $m_{a,i}h_{a,i} + Q_{rej} - Q_e = m_{a,o}h_{a,o}$ (2)
3) Mass conservation of condenser water: $m_{w,i} - m_e + m_m = m_{w,o}$ (3)
4) Heat conservation of condenser water: $m_{w,i}T_{w,i}C_{pw} + Q_{rej} + m_m T_m C_{pw} = m_{w,o}T_{w,o}C_{pw}$ (4)

In the governing equations, there are nine known parameters including: six input variables, $h_{a,i}$, $m_{a,i}$, $m_{w,i}$, m_m, T_m, $T_{w,i}$; a constant C_{pw}; and two measurable output variables $m_{w,o}$; $T_{w,o}$, and five unknowns: three output variables: $h_{a,i}$, $m_{a,o}$ and , Q_{arej} ; and two unmeasurable variables m_e and Q_e . As the unknown variables are more than the number governing equations, it is insufficient to determine all outlet conditions by the four governing equations alone, additional equations that could depict the characteristics of the cooling tower should be added. In Braun's model with effectiveness coefficient (1989), the derivative of saturation air enthalpy with respect to temperature, Cs, is introduced and used to formulate the cooling load model. ε_a is also added as a ratio of the actual heat transfer amount to the theoretical maximum amount:

$$Q_{rej} = \varepsilon_a m_a (h_{s,w,i} - h_{a,i}) \quad (5)$$

Analogous to a dry counter flow heat exchanger, the effectiveness, ε_a , is evaluated by :

$$\varepsilon_a = \frac{1-e^{-NTU\ \|1-m^*\|}}{1-m^* e^{-NTU\ \|1-m^*\|}} \quad (6)$$

with NTU, m*, and C_s calculated respectively by:

$$NTU = c\left|\frac{m_a}{m_w}\right|^{-(1+n)} \quad (7)$$

$$m^* = \frac{m_a}{m_w}\cdot\frac{C_s}{C_{pw}} \quad (8)$$

$$C_s = \left|\frac{dh_s}{dT}\right|_{T=T_w} \approx \frac{h_{s,w,i}-h_{s,w,o}}{T_{w,i}-T_{w,o}} \quad (9)$$

where, c and n are empirical constants specific to a particular tower design derived from the manufacturer. These two parameters are correlated as a straight line on a log-log plot of NTU vs. the flow rate ratio. Since C_s depends on outlet conditions of cooling tower— $T_{w,o}$ and $h_{s,w,o}$, it cannot be computed directly. Consequently, the outlet conditions of cooling tower need to be guessed initially at the reasonable values, and iterative computation is engaged for Equation (1)-(9) to calculate the ultimate results.

Although Braun's model is more accurate than Merkel's one, it also has several problems.

- The computations are very complicated, it needs iterative computation to get the final results, and the estimated outlet water temperature is needed before calculation;
- It is hard to find the function derivatives, which are useful in real-time optimization analysis;
- The model was derived based on mechanical principles, it only suitable for the counter flow cooling towers. For the crossflow cooling towers, a different model is needed.

II. MODELS AND ANALYSIS

The main difficulties in real-time application of Braun's model are the initial estimation of C_s and highly nonlinearities of ε_a , which resulted in a complicated and time consuming computation. To develop an effective engineering model, let's analysis both C_s and ε_a form fundamental laws of mass and energy balance.

2.1 Analysis Cs :

In Braun's model (1989), a straight line between water inlet temperature and water outlet temperature on the air saturation enthalpy with respect to temperature is used to approximate the curve between water inlet point and water outlet point (Fig 1), where C_s is the ratio of length of line (1) $h_{s,w,i} - h_{s,w,o}$ to line (2) $T_{w,i} - T_{w,o}$.

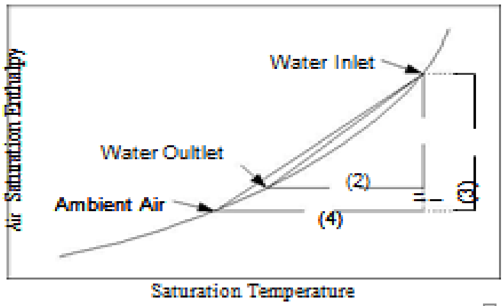

Fig 1. Saturation air enthalpy vs. temperature

For control and optimization purpose, however $T_{w,o}$ and $h_{s,w,o}$ are output variables, which need to be controlled, therefore, these two variables should not be used as input variables to calculate the heat rejection ration . Instead, we may express Equation (9) with measurable variables as:

$$C_s \left|\frac{dh_s}{dT}\right|_{T=T_w} \approx \frac{(h_{s,w,i}-h_{s,wb})+\int 1(\Delta h)}{(T_{w,i}-T_{wb})+\int 2(\Delta T)} \qquad (10)$$

where, ∆T is the approach of the cooling tower and represents the difference between line (4) and line (2) in Fig 1; ∆h is the saturated air enthalpy difference with respect to ∆T. By energy and mass conservation laws, the approach, ∆T, is a function of $\left|\frac{m_a}{m_w}\right|$ and $(T_{w,i} - T_{wb})$, as the approach is affected by the mass flow rate of both air and water and the temperature difference between inlet water and ambient air. ∆h can be considered as a function of ∆T, also the function of $\left|\frac{m_a}{m_w}\right|$ and $y = (T_{w.i} - T_{wb})$. Therefore, C_s can be described as:

$$C_s = f_3 \left|\left|\frac{m_a}{m_w}\right| (T_{w.i} - T_{wb})\right| \qquad (11)$$

2.2 Analysis ε_a:

From the Equation (6), (7), and (8), it clear shows that the heat transfer effectiveness, ε_a, is the function of NTU and m*, where NTU is the function of $\left|\frac{m_a}{m_w}\right|$ and m* is the function of $\left|\frac{m_a}{m_w}\right|$ and Cs. By Equation (11), Cs is the function of $\left|\frac{m_a}{m_w}\right|$ and $(T_{w.i} - T_{wb})$. Then, we can obtain a general expression for ε_a as:

$$\varepsilon_a = function \left|\left|\frac{m_a}{m_w}\right| (T_{w.i} - T_{wb})\right| = function(x, y) \qquad (12)$$

Where $x = \left|\frac{m_a}{m_w}\right|$ and $y = (T_{w.i} - T_{wb})$ The heat transfer effectiveness is the function of two variables, which are the inlet conditions of the cooling tower. As finding the exact function for Equation (12) is neither practical nor necessary for real-time application, the following engineering solution is proposed.

Engineering model:

In order to solve the problem above, Taylor's series expansion is used as an approximation of the unknown function in Equation (12). It is clear that ε_a is a continuous variable under normal operating conditions, its derivative and high-order derivatives exist. Thus, we can apply Taylor's series expansion for two variables into ε_a function. Because the characteristics of cooling towers are highly nonlinear, second-order Taylor's series expansion is used to better reflect the nonlinearity.

$f(x,y) = f(x_0,y_0) + \left|\frac{\partial f(x_0,y_0)}{\partial x}(x - x_0) + \frac{\partial f(x_0,y_0)}{\partial y}(y -\right.$
y0)+12!∂2f(x0,y0)∂x2x−x02+∂2f(x0,y0)∂y2y−y02 +2∂2f(x0,y0)∂x∂y(x−x0)y−y0
(13)

Where, (x_0, y_0) is any reasonable operating point of cooling tower near (x, y). Once the point (x_0, y_0) is determined, x_0, $y_0, f(x_0, y_0)$,$\frac{\partial f(x_0,y_0)}{\partial x}$, $\frac{\partial f(x_0,y_0)}{\partial y}$, $\frac{\partial^2 f(x_0,y_0)}{\partial x^2}$, $\frac{\partial^2 f(x_0,y_0)}{\partial y^2}$ and $\frac{\partial^2 f(x_0,y_0)}{\partial x \partial y}$ can be treat as the constants

To express the equation in neat way, Equation (13) is rearranged and written as a function of two variables form.

$$\varepsilon_a = c_0 + Cc_1\left|\frac{m_a}{m_w}\right| + c_2(T_{w.i} - T_{wb}) + c_3\left|\frac{m_a}{m_w}\right|^2 + c_4(T_{w.i} - T_{wb})^2 + c_5\left|\frac{m_a}{m_w}\right|(T_{w.i} - T_{wb}) \qquad (14)$$

Where, the coefficients, $c_0 - c_5$, are constants, and determined only by the cooling tower characteristics, which depend on the towers' structure and design.

III. ALGORITHMS FOR DETERMINING ENGINEERING MODEL

The real performance data of the cooling tower provided by manufacturers are used in our method. .The objective function is given as:

$min \sum_{i=1}^{N} \frac{1}{2}(function(c_0, c_1, c_2, c_3, c_4, c_5) -$
Fdatai)2 (15)

Where the function(.) is the right hand side of Equation (14) and the real performance data of cooling tower are represented by Fdatai. N is the number of the sampling points. Fdatai can be derived from manufacturers' data by lookup -table or interpolation. In order to obtain accurate results, the number of sampling points must more than that of coefficients, i.e. N> 5. Furthermore, the sampling points should be distributed evenly among the whole range of operation.

Nonlinear least square method for curve fitting is used to solve Equation (15), both Gauss-Newton and Levenberg-Marquardt methods are implemented in the optimization algorithms (Coleman et al. 1999). In Gauss- Newton method, a search direction dk is obtained at each major iteration step. The search direction is expressed as:

$$J\left(u_k\right)^T J\left(u_k\right)d_k = -J\left(u_k\right)F\left(u_k\right) \qquad (16)$$

Where, $u = [c_0, c_1, c_2, c_3, c, c_5,]^T$; $u_k$is the u value of the kth iteration;

$F_i(u_k) = \frac{1}{2}(function(u) - Fdata_i)^2$;

$F(u_k) = [F_1(u_k), F_2(u_k) \cdots, F_N(u_k)]^T$;

$$J(u_k) = \mathrm{j}\left\|\begin{matrix} c_0 \\ c_1 \\ . \\ . \\ . \\ c_5 \end{matrix}\right\| = \begin{vmatrix} \frac{\partial F_1}{\partial c_0} & \frac{\partial F_1}{\partial c_1} & \cdots & \frac{\partial F_1}{\partial c_5} \\ \frac{\partial F_2}{\partial c_0} & \frac{\partial F_2}{\partial c_1} & \cdots & \frac{\partial F_2}{\partial c_5} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F_N}{\partial c_0} & \frac{\partial F_N}{\partial c_1} & \cdots & \frac{\partial F_N}{\partial c_5} \end{vmatrix}$$

$J\left(u_k\right)$ is the Jacobian matrix with respect to u_k. In the case of $H(u_k)$ (Hessian Matrix of $Fi(u_k)$) is significant,Levenberg-Marquardt method is adopted. It uses a search direction between the Gauss- Newton direction and the steepest descent. This makes it less effective but more robust than the Gauss-Newton method. The Levenberg- Marquardt method is given by:

$$(J(u_k)^T J(u_k) + \lambda_k I)d_k = -J(u_k)F(u_k) \qquad (17)$$

In this equation, λ_k controls both magnitude and direction of d_k . When λ_k is zero, the direction d_k is identical to that of the Gauss-Newton method. As λ_k tends to infinity, d_k tends towards a vector of zeros and a steepest descent direction.

Remarks

1. In this method, the coefficients $c_0 - c_5$ are determined offline by curve fitting in the whole operating range. Therefore, the real-time output calculation is straightforward once the input variables are measured.
2. For more accurate results, it is possible to construct a look up table for coefficients $c_0 - c_5$ by dividing the whole operating range into sub-regions. One set of coefficients is selected at one time according to the cooling tower operation conditions.
3. In Bruan's model, both NTU and $\varepsilon_a$are exponential functions, which requires substantial computing effort. Whereas, in the new model ε_a is in a polynomial form which is much easier to calculate and suitable for on-line optimization.
4. For crossflow cooling towers, the analysis is almost same except Equation (6), which takes the following form according to the heat exchange principle.

$$\varepsilon_a = \left\|\frac{1}{m^*}\right\| \left\|1 - e^{-m^*\|-e^{-NTU}\|}\right\| \qquad (18)$$

However, this change will not affect the model structure. The differences of the different cooling tower models are determined by coefficients of Equation (14). Therefore, both counterflow and crossflow cooling towers can be represented by the same model.

5. In practice, it is very hard to measure the inlet and outlet airflow rate (ma,i and ma,o) accurately. This problem could be solved as follows:

- Using energy conservation principle, we can replace $m_a\left(h_{s,w,i} - h_{a,i}\right)$ by $m_w C_{pw}\left(T_{w,i} - T_{wb}\right)$;
- Writing

$$\varepsilon_a = \frac{m_{w,i}C_{pw}\,T_{w,i} + m_m\,C_{pw}\,T_m - m_{w,o}C_{pw}\,T_{w,o}}{m_{w,o}C_{pw}\,(T_{w,i} - T_{wb})} \qquad (19)$$

in Equation (19), according to the known variables: ε_a , $T_{w,i}$, T_{wb} , and $m_{w,o}$, Equation (14) is again used inversely to find the value of the mass airflow rate, $\dot{m}_a$. The value will then be employed to determine the overall heat rejection rate at the next sample time.

IV. RESULT

To validate the proposed model, the outputs of new model are compared with model provided by Braun (1989). Both counterflow and crossflow design cooling towers are used to illustrate its universeness. The parameters of cooling tower are given in following:

Air flow rate: 10.7-32.7 kg/s (1.41-4.32*10^5gpm);
Water flow rate: 21.7 kg/s (344 gpm);
Inlet water temperature: 38°C (100.4°F);
Ambient dry-bulb temperature: 35°C (95°F);
Ambient wet-bulb temperature: 21-31°C (69.8-87.8°F);
*c*in Equation (7): 2.3 (dimensionless);
*n*in Equation (7): -0.72(dimensionless).

For the counterflow cooling tower, the heat transfer effectiveness, ε_a ,varied with mass flow ratio of air to water and the ambient wet-bulb temperature shown in fig 2.

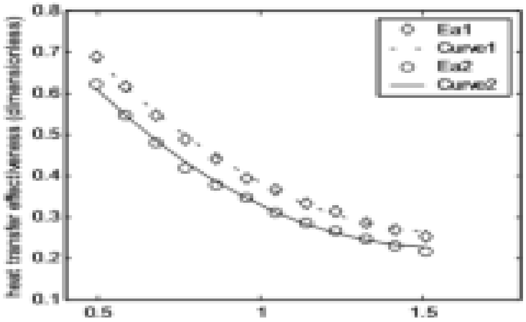

Fig 2. Comparison of heat transfer effectiveness of two models

where E_{a1} and E_{a2} are heat transfer effectiveness of Braun's model in ambient wet-bulb temperature 26°C and 30°C respectively. Curve1 is the heat transfer effectiveness given by the new model under 26°C wet-bulb temperature, and Curve2 under 30°C wet-bulb temperature. According to the figure, the results of two models are almost same.

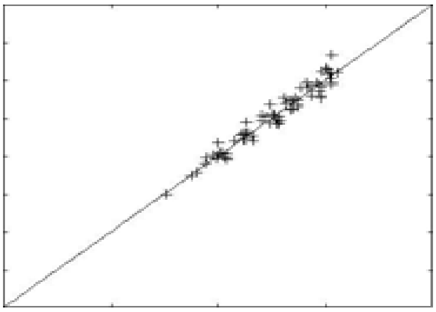

Fig 3. Counterflow cooling tower models

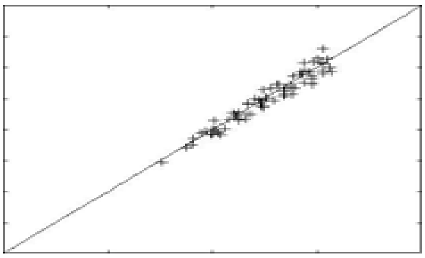

Fig 4. Counterflow cooling tower models

The results of heat rejection for the counterflow and crossflow cooling towers are shown in fig 3 and 4 respectively. There are totally 121 points on each figure. From the figures, it is clearly that the new model can predict the performance of cooling tower very well.

V. CONCLUSIONS

The new engineering model for cooling towers, which can be used to formulate both counterflow and crossflow cooling towers, has been presented in this paper. The methods of Merkel and Braun and fundamental laws of mass and energy balance are used to develop the effectiveness of heat exchange with polynomial form. Nonlinear least square curve - fitting methods are used to determine the coefficients of the model. Some engineering considerations are also discussed. The comparison study of existing and the new model is given to show that the new model can predict the performance of both counterflow and crossflow cooling tower accurately with less computation. As the manufacture data are used to determine the coefficients for the model, it is predicted that it should have better performance compared with the existing one's.

In practice, many unpredictable factors affect the performance of the cooling towers, such as outdoor airflow rate, interior problems of cooling tower, and measurement errors, etc. Therefore, the coefficients of cooling tower model may not be constant during the operational life span. Fault detection or adaptive scheme should be added to accommodate these changes; these aspects are also subject to future study. Utilizing the model for on-line optimization of both cooling towers and chillers as well as condenser water loop for HVAC systems is currently under study; the research results will be published soon.

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